Mode Localization Phenomena
in Flexibly Coupled Two Rotors

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Abstract

Rotating machineries are one of the most widely used elements in mechanical systems. Objective of this research is to investigate some nonlinear phenomena in rotor systems. First, we propose a general method to theoretically analyze nonlinear phenomena. The averaged equations for the complex amplitudes of the forward and backward whirling motions are derived in order to perform bifurcation analysis. We focus on the motion of a horizontally supported single-span rotor and theoretically and experimentally clarify that the cubic and quintic nonlinearities take important role for the nonlinear dynamics of the system. Furthermore, it is theoretically and experimentally shown that this system exhibits hardening and softening types responses depending on the rotational speed, due to the effects of gravity and nonlinear characteristics of the rotor. Next, we consider multiple-span rotors. Mode localizations in a weakly coupled two-span rotor system are theoretically investigated. One rotor has a slight unbalance and the other one is well-assembled. First, the equations governing the whirling motions of the coupled rotors are derived by taking into account nonlinearity in each span and weakness of the coupling between them. The averaged equations indicate that the nonlinear normal modes are bifurcated from the linear normal modes. Also, it is theoretically clarified that whirling motion caused by the unbalance in the rotor is localized in the rotor with unbalance or in the other rotor without unbalance.
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Chapter 1

Introduction

Rotating machineries, such as steam turbines, gas turbines, motors and so on, are one of the most widely used elements in mechanical systems. However, the rotating parts of such machineries often become main source of vibrations. Hence, detecting the source and analyzing the feature of the vibrations are the critical issue, in order to enhance the stability and the reliability of mechanical systems. In many systems, such as power plant and jet engine, the rotating machinery consists of multiple-span rotors, which are supported by multiple bearings.

Multi-degree of freedom nonlinear systems have generally nonlinear normal modes [1] whose number exceeds the degree of freedom of the systems. This nonlinear feature is concerned with bifurcations of the mode and also the bifurcations can cause another interesting phenomenon, i.e., mode localization. Then, a subclass of nonlinear normal modes is spatially confined to a certain areas in the system. It is known for rectilinear systems that such mode localizations are caused from the existence of weekly coupling and nonlin-
earity. There have been many studies on nonlinear normal modes and mode localizations over the past few decades. Vakakis theoretically clarified that the mode localization in the multi-span beam is produced by means of the geometric nonlinearity of the beams and the weak coupling with torsional stiffeners [2]. For a two-span beam, some experimental results confirmed the production of nonlinear normal modes [3]. The studies on nonlinear normal modes in the systems under the external excitation and parametric excitation are also attractive from many researchers [4, 5]. The new methods for accurately analyzing the nonlinear normal modes and mode localizations are proposed based on invariant manifold theory [6] and Galerkin-based approach [7]. However, all studies on nonlinear normal modes are related to rectilinear oscillatory systems. There have been no reports on multiple-span rotors systems to our knowledge.

In the present study, we consider a two-span rotor system and investigate the mode localization phenomena. Generally the coupling between each rotor in the multiple-span system has very week bending stiffness so that the whirling motion in a certain span does not excite the other spans strongly. Therefore, it seems that the multiple-span rotors system has representative nonlinear normal modes and easily causes mode localizations.

In this study, first, we propose a general method to theoretically analyze nonlinear phenomena in rotor systems. The averaged equations governing the complex amplitudes of the forward and backward whirling motions are derived to perform bifurcation analysis. Furthermore, we discuss in detail a horizontally supported single-span rotor taking into account cubic and quintic nonlinearities. We theoretically and experimentally show the occurrences of hardening and softening types responses depending on the rotational speed, due to the
effects of gravity and nonlinear characteristics of the rotor. After discussing the nonlinear effect on dynamics of a single-span rotor system, we consider multiple-span rotors. We derive the equations governing the whirling motion and apply the method of multiple scales to obtained averaged equations. The obtained averaged equations present different kinds of mode localization phenomena.
Chapter 2

Method of Nonlinear Analysis for Rotor System

2.1 Equations of motion of single-span rotor system

2.1.1 Analytical model

Figure 2.1: Analytical model (single-span rotor system)
We consider a Jeffcott rotor as shown in Fig. 2.1(a). A rigid disk is mounted at the mid-span of a massless elastic shaft which is supported at both ends with ball bearings. We introduce the static coordinate system as shown in Fig. 2.1(b). The origin O of the coordinate system O-xy coincides with the bearing centerline connecting the centers of the right and left bearings. The disk of the rotor has mass $m$ and its center of gravity $G$ deviates slightly $(e_d)$ from the geometrical center $M$. The planer motion of the disk can be expressed by the displacement of the point $M$ from the origin $O$. Furthermore, assuming the cubic nonlinearity in the stiffness of the bearings and shaft, which is the most fundamental symmetric nonlinearity [8], the equations of motion of the rotor system can be written as follows:

$$m\frac{d^2x}{dt^2} + c_d\frac{dx}{dt} + kx + \beta_{sd}(x^2 + y^2)x = me_d\omega^2 \cos \omega t \quad (2.1)$$

$$m\frac{d^2y}{dt^2} + c_d\frac{dy}{dt} + ky + \beta_{sd}(x^2 + y^2)y = me_d\omega^2 \sin \omega t - mg_d, \quad (2.2)$$

where $\omega$, $c_d$, $k$, $\beta_{sd}$ and $g_d$ are the angular velocity of the shaft, the viscous damping coefficient, the linear spring constant of the elastic shaft, the cubic nonlinear spring constant and the gravity acceleration, respectively.

### 2.1.2 Dimensionless equations of motion

The length and time are normalized using the length of a span $l$ and the inverse of linear natural frequency of a span $1/\sqrt{k/m}$. We denote the dimensionless quantities of $t$, $x$ and
y with $t^*$, $x^*$ and $y^*$, respectively. Hence, we obtain the dimensionless equations as follows:

\begin{align}
\ddot{x} + c\dot{x} + x + \beta_3(x^2 + y^2)x &= e\nu^2 \cos \nu t \tag{2.3} \\
\ddot{y} + c\dot{y} + y + \beta_3(x^2 + y^2)y &= e\nu^2 \sin \nu t - g, \tag{2.4}
\end{align}

where the dimensionless parameters, $e$, $c$, $\beta_3$, $g$ and $\nu$, are expressed as follows:

\begin{align*}
e &= \frac{c_d}{l}, \\
c &= \frac{c_d}{\sqrt{mk}}, \\
\beta_3 &= \frac{\beta_3d^2}{k}, \\
g &= \frac{mg_d}{kl}, \\
\nu &= \frac{\omega}{\sqrt{k/m}}.
\end{align*}

The dot denotes the derivative with respect to dimensionless time $t^*$. In Eqs. (2.3) and (2.4) and hereafter, the asterisk is omitted.

### 2.1.3 Transformation into complex form

Next, we rewrite the motion of the system by using complex vector representation as follows [9]:

\begin{equation}
z = x + iy. \tag{2.5}
\end{equation}

Then, the dimensionless equation of motion is transformed as follows:

\begin{equation}
\ddot{z} + c\dot{z} + z + \beta_3|z|^2z + ig = e\nu^2e^{i\nu t}. \tag{2.6}
\end{equation}

The transformation is an essential process to separately obtain the averaged equations of the forward and backward whirling modes.
2.2 Theoretical analysis

2.2.1 Averaged equation

In this section, we derive averaged equations from the dimensionless equation of motion Eq. (2.6) by using the method of multiple scales [10]. First, we perform the scaling of some parameters as

\[ c = \epsilon^2 \hat{c}, \quad g = \epsilon \hat{g}, \quad e = \epsilon^3 \hat{e}, \]

where (\( \hat{\cdot} \)) denotes “of order \( O(1) \)” and \( \epsilon(|\epsilon| << 1) \) is a bookkeeping device. Then, the dimensionless equation of motion is

\[
\ddot{z} + \epsilon^2 \hat{c} \dot{z} + z + \beta_3 |z|^2 z + i \epsilon \hat{g} = \epsilon^3 \hat{e} \nu^2 e^{i \nu t}.
\]

(2.7)

We seek the approximate solution of Eq. (2.7) in the form

\[
z = \epsilon z_1(t_0, t_2) + \epsilon^3 z_3(t_0, t_2) + \cdots,
\]

(2.8)

where \( t_0 = t \) is the fast scale and \( t_2 = \epsilon^2 t \) is the stretched time scale.

Also, to express quantitatively the nearness of the rotational speed \( \nu \) to the natural frequency of the rotor, we introduce a detuning parameter \( \sigma \) defined by

\[
\nu = 1 + \sigma = 1 + \epsilon^2 \hat{\sigma}.
\]

(2.9)
Substituting Eq. (2.8) into Eq. (2.7) and equating coefficients of like powers of $\epsilon$ yields

- $O(\epsilon)$

$$D_0^2 z_1 + z_1 = -i\hat{g} \quad (2.10)$$

- $O(\epsilon^3)$

$$D_0^2 z_3 + z_3 = -2D_0 D_2 z_1 - \hat{c} D_0 z_1 - \beta_3 |z_1| z_1 + \epsilon e^{i\nu t_0}, \quad (2.11)$$

where $D_i = \partial/\partial t_i$.

The solutions of Eq. (2.10) can be written as

$$z_1 = A_f e^{it_0} + A_b e^{-it_0} - i\hat{g}, \quad (2.12)$$

where $A_f$ and $A_b$ are complex amplitudes of the forward and backward whirls of the rotor, respectively. These complex amplitudes are varied with slow time scale $t_2$.

Substituting Eq. (2.12) into Eq. (2.11) gives

$$D_0^2 z_3 + z_3 = \{ -2iD_2 A_f - i\hat{c} A_f - \beta_3 (|A_f|^2 A_f + 2|A_b|^2 A_f + \hat{g}^2 A_f - \hat{g}^2 A_b) + \epsilon e^{i\nu t_2} \} e^{it_0}$$

$$+ \{ 2iD_2 A_b + i\hat{c} A_b - \beta_3 (|A_b|^2 A_b + 2|A_f|^2 A_b - \hat{g}^2 A_f + \hat{g}^2 A_b) \} e^{-it_0} + N.S.T. \quad (2.13)$$

where N.S.T. denotes terms not to proportional to $e^{it_0}$ or $e^{-it_0}$. The condition not to
produce the secular term proportional to $e^{i\omega t_0}$ in the solution of $z_3$ is

$$2iD_2A_f + i\dot{c}A_f + \beta_3(|A_f|^2A_f + 2|A_b|^2A_f + 2\hat{g}^2A_f - \hat{g}^2A_b) - \dot{c}e^{i\omega t_2} = 0. \quad (2.14)$$

Also, the condition not to produce the secular term proportional to $e^{-i\omega t_0}$ is

$$2iD_2A_b + i\dot{c}A_b - \beta_3(|A_b|^2A_b + 2|A_f|^2A_b - \hat{g}^2A_f + 2\hat{g}^2A_b) = 0. \quad (2.15)$$

Generally in the case of rectilinear systems the complex conjugate of the solvability condition Eq. (2.14) is equivalent to the solvability condition Eq. (2.15). One of them is needed to obtain the averaged equation. However, contrast with the rectilinear systems, these conditions are generally independent and the first and second conditions lead to averaged equations of the forward and backward whirling modes, respectively.

First, we consider the case without gravity ($g = 0$). The averaged equation for the backward whirl Eq. (2.15) is

$$2iD_2A_b + i\dot{c}A_b - \beta_3(|A_b|^2 + 2|A_f|^2)A_b = 0. \quad (2.16)$$

Subtracting the equation multiplying the complex conjugate of Eq. (2.16) by $A_-$, from the equation multiplying Eq. (2.16) by $\bar{A}_-$, yields the following equation:

$$\frac{d|A_b|^2}{dt} = -c|A_b|^2. \quad (2.17)$$

Therefore, the amplitude of the backward whirling mode decays to zero with time.
Next, we examine whirling motion in the case with gravity. Substituting

\[ A_f = a_f e^{i(\varphi_f + \dot{\varphi}_f t)} \]  \hspace{1cm} (2.18) \]

\[ A_b = a_b e^{i(\varphi_b - \dot{\varphi}_b t)} \]  \hspace{1cm} (2.19) \]

into Eqs. (2.14) and (2.15), and separating real and imaginary parts, we obtain the following averaged equations expressing the slow time scale modulations of the amplitudes and phases of forward and backward whirls:

\[ \frac{d a_f}{dt} = -\frac{1}{2} \alpha a_f - \frac{1}{2} \beta_g^2 a_b \sin(\varphi_f + \varphi_b) - \frac{1}{2} \epsilon \sin \varphi_f \]  \hspace{1cm} (2.20) \]

\[ a_f \frac{d \varphi_f}{dt} = -\sigma a_f + \frac{1}{2} \beta a_f^3 + \beta a_f a_b^2 + \beta g^2 a_f - \frac{1}{2} \beta g^2 a_b \cos(\varphi_f + \varphi_b) - \frac{1}{2} \epsilon \cos \varphi_f \]  \hspace{1cm} (2.21) \]

\[ \frac{d a_b}{dt} = -\frac{1}{2} \alpha a_b + \frac{1}{2} \beta g^2 a_f \sin(\varphi_f + \varphi_b) \]  \hspace{1cm} (2.22) \]

\[ a_b \frac{d \varphi_b}{dt} = \sigma a_b - \frac{1}{2} \beta a_b^3 - \beta a_f^2 a_b - \beta g^2 a_b + \frac{1}{2} \beta g^2 a_f \cos(\varphi_f + \varphi_b), \]  \hspace{1cm} (2.23) \]

where \( a_f = \epsilon \dot{a}_f \) and \( a_b = \epsilon \dot{a}_b \). By using Eqs. (2.8), (2.9), (2.12), (2.18) and (2.19), the approximate solution of Eq. (2.6) can be expressed as

\[ z = a_f e^{i(\nu t + \varphi_f)} + a_b e^{i(-\nu t + \varphi_b)} - ig + O(\epsilon^3). \]  \hspace{1cm} (2.24) \]

The slow time variations of \( a_f, \varphi_f, a_b \) and \( \varphi_b \) are governed with Eqs. (2.20)-(2.23).
2.2.2 Frequency response curves

We consider the conditions in Eqs. (2.20)-(2.23) in the steady states. Furthermore, examining their stabilities by these equations [11], we obtain frequency response curves as Figs. 2.2 and 2.3, where $c = 1.2 \times 10^{-2}, \beta_3 = 2.52 \times 10^3, \epsilon = 3.85 \times 10^{-5}$. The solid and dashed lines denote stable and unstable steady state amplitude, respectively. In the case when the rotor does not experience the gravity effect, only forward whirling motion is produced as shown in Fig. 2.2. On the other hand, in the case when the rotor experiences the gravity effect, both forward and backward whirling motions are produced as shown in Fig. 2.3, due to the effects of gravity and nonlinear characteristics of the rotor.
Figure 2.2: Theoretical frequency response curve (without gravity, $g = 0.00$)

Figure 2.3: Theoretical frequency response curve (with gravity, $g = 3.46 \times 10^{-3}$)
Chapter 3

Analysis of Horizontally Supported Single-Span Rotor System

3.1 Equations of motion of single-span rotor system

3.1.1 Analytical model

(a) Jeffcott rotor

(b) Coordinate system

Figure 3.1: Analytical model (single-span rotor system)
In this chapter, we consider a horizontally supported rotor system with cubic and quintic
nonlinearities as Fig. 3.1. Then, the rotor experiences the gravity effect in the negative
direction of \( y \) axis. Namely we examine the following equations of motion:

\[
\begin{align*}
md\frac{d^2x}{dt^2} + c_{xd}\frac{dx}{dt} + kx + \beta_{3d}(x^2 + y^2)x + \beta_{5d}(x^2 + y^2)^2x &= me_d\omega^2\cos\omega t \quad (3.1) \\
md\frac{d^2y}{dt^2} + c_{yd}\frac{dy}{dt} + ky + \beta_{3d}(x^2 + y^2)y + \beta_{5d}(x^2 + y^2)^2y &= me_d\omega^2\sin\omega t - mg, \quad (3.2)
\end{align*}
\]

where \( \omega, c_{xd}, c_{yd}, k, \beta_{3d}, \beta_{5d} \) and \( g_d \) are the angular velocity of the shaft, the viscous
damping coefficient in the horizontal direction, the viscous damping coefficient in the vertical
direction, the linear spring constant of the elastic shaft, the cubic nonlinear spring constant,
the quintic nonlinear spring constant and the gravity acceleration, respectively.

### 3.1.2 Motions about equilibrium position

We denote by \( y \) the displacement of the mass from the unstretched spring position. Be-
because of gravity, the equilibrium position differs from the bearing centerline by the static
displacement \( y_{st} \). Denoting the equilibrium point by \( y = y_{st} \) and considering Eq. (3.2), we
conclude that the equilibrium positions must satisfy the equation

\[
k_{y_{st}} + \beta_{3d}y_{st}^3 + \beta_{5d}y_{st}^5 = -mg. \quad (3.3)
\]

The solution yields the equilibrium positions \( y_{st} \).

Inserting \( y = y_{st} + \Delta y \) in Eqs. (3.1) and (3.2), we obtain the differential equations of
motions in $x$ and $\Delta y$ are:

$$m \ddot{x} + c_{xd} \dot{x} + \omega_{xd}^2 x + f_{1xd} \Delta y x + f_{2xd} \Delta x^3 + f_{3xd} \Delta y^2 = m e_d \omega^2 \cos \omega t$$  \hspace{1cm} (3.4)$$

$$m \Delta \ddot{y} + c_{yd} \Delta \dot{y} + \omega_{yd}^2 \Delta y + f_{1yd} \Delta y^2 + f_{2yd} \Delta x^2 \Delta y + f_{3yd} \Delta y^3 = m e_d \omega^2 \sin \omega t,$$

(3.5)

where

$$\omega_{xd}^2 = k + \beta_3 d y_{st}^2 + \beta_5 d y_{st}^4$$

$$f_{1xd} = 2 \beta_3 d y_{st} + 4 \beta_5 d y_{st}^3$$

$$f_{2xd} = \beta_3 d + 2 \beta_5 d y_{st}^2$$

$$f_{3xd} = \beta_3 d + 6 \beta_5 d y_{st}^2$$

$$\omega_{yd}^2 = k + 3 \beta_3 d y_{st}^2 + 5 \beta_5 d y_{st}^4$$

$$f_{1yd} = \beta_3 d y_{st} + 2 \beta_5 d y_{st}^3$$

$$f_{2yd} = 3 \beta_3 d y_{st} + 10 \beta_5 d y_{st}^3$$

$$f_{3yd} = \beta_3 d + 6 \beta_5 d y_{st}^2$$

$$f_{4yd} = \beta_3 d + 10 \beta_5 d y_{st}^2.$$. 
3.1.3 Dimensionless equations of motion

The length and time are normalized using the static displacement $y_{st}$ and the inverse of linear natural frequency of a span $1/\sqrt{k/m}$. We set the dimensionless parameters as follows:

$$t = \sqrt{\frac{m}{k}} t^*, \quad x = y_{st} x^*, \quad \Delta y = y_{st} \Delta y^*.$$

Hence, we obtain the following dimensionless equations:

$$\ddot{x} + c_x \dot{x} + \omega_x^2 x + f_{1x} x \Delta y + f_{2x} x^3 + f_{3x} x \Delta y^2 = e \nu^2 \cos \nu t \quad (3.6)$$

$$\Delta \dot{y} + c_y \Delta \dot{y} + \omega_y^2 \Delta y + f_{1y} x^2 + f_{2y} \Delta y^2 + f_{3y} x^2 \Delta y + f_{4y} \Delta y^3 = e \nu^2 \sin \nu t, \quad (3.7)$$

where the dimensionless parameters, $e$, $c$, $\omega$, $f$ and $\nu$, are expressed as follows:

$$e = \frac{e_d}{y_{st}}, \quad c_x = \frac{c_{xd}}{\sqrt{mk}}, \quad c_y = \frac{c_{yd}}{\sqrt{mk}}, \quad \omega_x = \frac{\omega_{xd}}{\sqrt{k}}, \quad \omega_y = \frac{\omega_{yd}}{\sqrt{k}},$$

$$f_{1x} = \frac{f_{1xd} y_{st}}{k}, \quad f_{2x} = \frac{f_{2xd} y_{st}^2}{k}, \quad f_{3x} = \frac{f_{3xd} y_{st}^2}{k},$$

$$f_{1y} = \frac{f_{1yd} y_{st}}{k}, \quad f_{2y} = \frac{f_{2yd} y_{st}^2}{k}, \quad f_{3y} = \frac{f_{3yd} y_{st}^2}{k}, \quad f_{4y} = \frac{f_{4yd} y_{st}^2}{k}, \quad \nu = \frac{\omega}{\sqrt{k/m}}.$$

The dot denotes the derivative with respect to dimensionless time $t^*$. In Eqs. (3.6) and (3.7) and hereafter, the asterisk is omitted.
3.2 Theoretical analysis

3.2.1 Averaged equation

In this section, we derive averaged equations from the dimensionless equations of motion, Eqs. (3.6) and (3.7), by using the method of multiple scales [10]. First, we perform the scaling of some parameters according to

\[ e = \epsilon^3 \hat{e}, \quad c = \epsilon^2 \hat{c}, \quad f = \hat{f}, \]

where (\( \hat{\cdot} \)) denotes “of order \( O(1) \)” and \( \epsilon(|\epsilon| < 1) \) is a bookkeeping device. We seek the approximate solutions of Eqs. (3.6) and (3.7) in the form

\[ x = \epsilon x_1 + \epsilon^2 x_2 + \epsilon^3 x_3 + \cdots \] \hspace{1cm} (3.8)
\[ \Delta y = \epsilon y_1 + \epsilon^2 y_2 + \epsilon^3 y_3 + \cdots. \] \hspace{1cm} (3.9)

We introduce the multiple time scales as follows:

\[ t_0 = t, \quad t_1 = \epsilon t, \quad t_2 = \epsilon^2 t. \]

Also, to express quantitatively the nearness of the rotational speed \( \nu \) to the natural frequency in the horizontal direction at the equilibrium point, we introduce the detuning parameter \( \sigma \) defined by

\[ \nu = \omega_x + \sigma = \omega_x + \epsilon^2 \hat{\sigma}. \] \hspace{1cm} (3.10)
Substituting Eqs. (3.8) and (3.9) into Eqs. (3.6) and (3.7) and equating coefficients of like powers of $\epsilon$ yields

- $O(\epsilon)$

\[
D_0^2 x_1 + \omega_x^2 x_1 = 0 \\
D_0^2 y_1 + \omega_x^2 y_1 = 0
\] (3.11) (3.12)

- $O(\epsilon^2)$

\[
D_0^2 x_2 + \omega_x^2 x_2 = -2D_0 D_1 x_1 - f_{1x} x_1 y_1 \\
D_0^2 y_2 + \omega_x^2 y_2 = -2D_0 D_1 y_1 - f_{1y} x_1^2 - f_{2y} y_1^2
\] (3.13) (3.14)

- $O(\epsilon^3)$

\[
D_0^2 x_3 + \omega_x^2 x_3 = -2D_0 D_1 x_2 - (D_1^2 + 2D_0 D_2) x_1 - \hat{c}_x D_0 x_1 \\
- f_{1x}(x_1 y_2 + x_2 y_1) - f_{2x} x_1^3 - f_{3x} x_1 y_1^2 + \hat{c}_x^2 \cos \nu t
\] (3.15)

\[
D_0^2 y_3 + \omega_x^2 y_3 = -2D_0 D_1 y_2 - (D_1^2 + 2D_0 D_2) y_1 - 2\omega_x \hat{\omega}_y - y_1 - \hat{c}_y D_0 y_1 \\
- 2f_{1y} x_1 x_2 - 2f_{2y} y_1 y_2 - f_{3y} x_1^2 y_1 - f_{4y} y_1^3 + \hat{c}_y^2 \sin \nu t,
\] (3.16)

where $D_i = \partial / \partial t_i$ and $\omega_- = \omega_y - \omega_x$ ($\omega_- = \sigma^2 \hat{\omega}_-$).
The solutions of Eqs. (3.11) and (3.12) can be written as

\[
x_1 = A_x(t_1, t_2)e^{i\omega_x t_0} + cc \tag{3.17}
\]
\[
y_1 = A_y(t_1, t_2)e^{i\omega_y t_0} + cc. \tag{3.18}
\]

Substituting Eqs. (3.17) and (3.18) into (3.13) leads to

\[
D_0^2 x_2 + \omega_x^2 x_2 = -2i\omega_x D_1 A_x e^{i\omega_x t_0} - f_{1x} (A_x^2 e^{2i\omega_x t_0} + A_x A_y e^{(i\omega_x - \omega_y)t_0} + A_y^2 e^{2i\omega_y t_0}) + cc. \tag{3.19}
\]

Any particular solution of Eq. (3.19) has a secular term containing the factor \( t_0 \exp(i\omega_x t_0) \) unless

\[
D_1 A_x = 0.
\]

Therefore \( A_x \) must be independent of \( t_1 \). With \( D_1 A_x = 0 \), the solution of Eq. (3.13) is

\[
x_2 = -f_{1x} \left\{ -\frac{A_x^2 + A_y^2}{3} e^{2i\omega_x t_0} + A_x A_y \right\} + cc. \tag{3.20}
\]

Similarly, the solution of Eq. (3.14) is

\[
y_2 = -\frac{f_{1y}}{\omega_x^2} \left\{ -\frac{A_x^2}{3} e^{2i\omega_x t_0} + |A_x|^2 \right\} - \frac{f_{2y}}{\omega_x^2} \left\{ -\frac{A_y^2}{3} e^{2i\omega_y t_0} + |A_y|^2 \right\} + cc. \tag{3.21}
\]
We substitute $x_1$, $y_1$, $x_2$ and $y_2$ from Eqs. (3.17), (3.18), (3.20) and (3.21) into Eqs. (3.15) and (3.16). The conditions to eliminate secular terms from $x_3$ and $y_3$ are

\[
2i\omega_x D_2 A_x + i\dot{c}_x \omega_x A_x - \left(\frac{5f_{1x}f_{1y}}{3\omega_x^2} - 3f_{2x}\right)|A_x|^2 A_x - \left(\frac{2f_{1x}f_{2y}}{\omega_x^2} + \frac{2f_{1x}^2}{3\omega_x^2} - 2f_{3x}\right)|A_y|^2 A_x \\
-\left(-\frac{f_{1x}f_{2y}}{3\omega_x^2} + \frac{f_{1x}^2}{\omega_x^2} - f_{3x}\right)A_y^2 A_x - \frac{1}{2}i(c_2^x\omega_x e^{i\sigma t_2} = 0 \quad (3.22)
\]

\[
2i\omega_x D_2 A_y + i\dot{c}_y \omega_x A_y + 2\omega_x \dot{\omega}_A y - \left(-\frac{10f_{2y}}{3\omega_x^2} - 3f_{4y}\right)|A_y|^2 A_y \\
-\left(\frac{4f_{1x}f_{1y}}{3\omega_x^2} + \frac{10f_{1x}f_{2y}}{3\omega_x^2} - 2f_{3y}\right)|A_x|^2 A_y - \left(\frac{2f_{1x}f_{1y}}{\omega_x^2} - f_{3y}\right)A_x^2 A_y + i\frac{1}{2}c_2^x\omega_x e^{i\sigma t_2} = 0. \quad (3.23)
\]

Substituting

\[
A_x = \frac{1}{2}c_2^{(t_2)}e^{i(\varphi_2(t_2) + \sigma t_2)} \quad (3.24)
\]

\[
A_y = \frac{1}{2}c_3^{(t_2)}e^{i(\varphi_2(t_2) + \sigma t_2)} \quad (3.25)
\]

into Eqs. (3.22) and (3.23), and separating real and imaginary parts, we obtain the following averaged equations expressing the modulations of the amplitudes and phases in each rotor:

\[
\frac{d}{dt}a_x = -\frac{1}{2}c_2 a_x - \frac{1}{2}e\omega_x \sin \varphi_x \quad (3.26)
\]

\[
a_x \frac{d}{dt} \varphi_x = -\sigma a_x + \left(-\frac{5f_{1x}f_{1y}}{24\omega_x^3} + \frac{3f_{2x}}{8\omega_x}a_x^3 \right)
+ \left(-\frac{5f_{1x}}{24\omega_x^3}(f_{1x} + f_{2y}) + \frac{3f_{3x}}{8\omega_x}a_x a_y^2 \right) - \frac{1}{2}e\omega_x \cos \varphi_x \quad (3.27)
\]

\[
\frac{d}{dt}a_y = -\frac{1}{2}c_y a_y - \frac{1}{2}e\omega_x \cos \varphi_y \quad (3.28)
\]

\[
a_y \frac{d}{dt} \varphi_y = -\sigma a_y + \omega - a_y + \left(-\frac{5f_{2y}}{12\omega_x^3} + \frac{3f_{4y}}{8\omega_x}a_y^3 \right)
+ \left(-\frac{5f_{1y}}{12\omega_x^3}(f_{1x} + f_{2y}) + \frac{3f_{3y}}{8\omega_x}a_x a_y^2 \right) + \frac{1}{2}e\omega_x \sin \varphi_y, \quad (3.29)
\]
where $a_x = \epsilon \hat{a}_x$ and $a_y = \epsilon \hat{a}_y$. By using Eqs. (3.8), (3.9), (3.10), (3.17), (3.18), (3.24) and (3.25), the approximate solutions Eqs. (3.6) and (3.7) can be expressed as

$$x = a_x \cos(\nu t + \varphi_x) + O(\epsilon^2) + O(\epsilon^3)$$

(3.30)

$$\Delta y = a_y \cos(\nu t + \varphi_y) + O(\epsilon^2) + O(\epsilon^3).$$

(3.31)

The slow time variations of $a_x$, $\varphi_x$, $a_y$, $\varphi_y$ are governed with Eqs. (3.26)-(3.29).

### 3.2.2 Frequency response curves

From Eqs. (3.26)-(3.29), we can investigate steady state amplitudes and their stability. Then, we obtain frequency response curves as Fig. 3.2, where $\epsilon = 2.4 \times 10^{-3}$, $c_x = c_y = 4.0 \times 10^{-3}$. The solid and dashed lines denote stable and unstable steady state amplitude, respectively. It is seen Fig. 3.2(a) that the response in the horizontal direction is a hardening-type one. The bending of the frequency response curves produces a jump phenomenon. When $\nu$ is near $\omega_x$, the amplitude is relatively large. On the other hand, the response in the vertical direction displayed in Fig. 3.2(b) is a softening-type one due to the effects of gravity and nonlinear characteristics of the rotor. When $\nu$ is near $\omega_y$, the amplitude is relatively large.

Figure 3.3 shows the frequency response curves in the case of $\epsilon = 3.12 \times 10^{-3}$, $c_x = 7.02 \times 10^{-3}$, $c_y = 3.17 \times 10^{-3}$, which correspond to those of the subsequent experiment. In the case of $c_x = c_y$, the peak of the amplitude in the $x$ direction is equal to one in the $y$ direction as shown in Fig. 3.2 ($k = 3.09 \times 10^4$ N/m, $\beta_{3d} = 1.58 \times 10^9$ N/m$^3$, $\beta_{5d} = -3.86 \times 10^{13}$ N/m$^5$).
On the other hand, in the case of $c_x \neq c_y$, the peak of the amplitude in the $x$ direction is not equal to one in the $y$ direction depending on the amount of damping as shown in Fig. 3.3.

Furthermore, Fig. 3.4 shows the frequency response curves of the rotor in the case when the nonlinear stiffness of the system is assumed only by cubic term ($k = 3.50 \times 10^4$ N/m, $\beta_{3d} = 6.99 \times 10^8$ N/m$^3$, $\beta_{5d} = 0$ N/m$^5$). The response in the vertical direction is not bend similar to the linear rotor systems. However, as mentioned later, the experimental result shows the bending frequency response curve.

![Graphs showing frequency response curves for rotor in x and y directions](image)

Figure 3.2: Frequency response curve ($e = 2.4 \times 10^{-3}$, $c_x = c_y = 4.0 \times 10^{-3}$, $\omega_x = 1.10$, $\omega_y = 1.25$, $f_{1x} = 3.58 \times 10^{-1}$, $f_{2x} = 1.79 \times 10^{-1}$, $f_{3x} = 7.94 \times 10^{-2}$, $f_{1y} = 1.78 \times 10^{-1}$, $f_{2y} = 4.37 \times 10^{-1}$, $f_{3y} = 7.94 \times 10^{-2}$, $f_{4y} = -2.02 \times 10^{-2}$)
Figure 3.3: Frequency response curve \( (\epsilon = 3.12 \times 10^{-3}, \ c_x = 7.02 \times 10^{-3}, \ c_y = 3.17 \times 10^{-3}, \ \omega_x = 1.10, \ \omega_y = 1.25, \ f_{1x} = 3.58 \times 10^{-1}, \ f_{2x} = 1.79 \times 10^{-1}, \ f_{3x} = 7.94 \times 10^{-2}, \ f_{1y} = 1.78 \times 10^{-1}, \ f_{2y} = 4.37 \times 10^{-1}, \ f_{3y} = 7.94 \times 10^{-2}, \ f_{4y} = -2.02 \times 10^{-2}) \)

Figure 3.4: Frequency response curve \( (\epsilon = 2.4 \times 10^{-3}, \ c_x = c_y = 4.0 \times 10^{-3}, \ \omega_x = 1.04, \ \omega_y = 1.12, \ f_{1x} = 1.71 \times 10^{-1}, \ f_{2x} = 8.56 \times 10^{-2}, \ f_{3x} = 8.56 \times 10^{-2}, \ f_{1y} = 8.56 \times 10^{-2}, \ f_{2y} = 2.57 \times 10^{-1}, \ f_{3y} = 8.56 \times 10^{-2}, \ f_{4y} = 8.56 \times 10^{-2}) \)
Chapter 4

Analysis of Two-Span Rotor System

4.1 Equations of motion of two-span rotor system

4.1.1 Analytical model

We consider a two-coupled Jeffcott rotors as shown in Fig. 4.1. A rigid disk is mounted at the mid-span of a massless elastic shaft which is supported at both ends with ball bearings.
We introduce the static coordinate system as shown in Fig. 4.2. The origins O₁ and O₂ of the coordinate systems O₁-x₁y₁ and O₂-x₂y₂ coincide with the bearing centerline connecting the centers of the right and left bearings for each shaft. The disk of the rotor 1 has mass \( m \) and its center of gravity G deviates slightly \( (e_d) \) from the geometrical center M. The disk of the rotor 2 has same mass, but the setup of rotor 2 is well-assembled. The planar motion of the disk can be expressed by the displacement of the point M from the origin O. Furthermore, assuming the cubic nonlinearity in the stiffness of the bearings and shaft, which is the most fundamental symmetric nonlinearity [8], the equations of motion of the two-span rotor system can be written as follows:

\[
\begin{align*}
mx_1'' + c_d x_1' + kx_1 + \gamma_d x_2 + \beta_3d(x_1^2 + y_1^2)x_1 &= me_d\omega^2 \cos \omega t \quad (4.1) \\
my_1'' + c_d y_1' + ky_1 + \gamma_d y_2 + \beta_3d(x_1^2 + y_1^2)y_1 &= me_d\omega^2 \sin \omega t \quad (4.2) \\
mx_2'' + c_d x_2' + kx_2 + \gamma_d x_1 + \beta_3d(x_2^2 + y_2^2)x_2 &= 0 \quad (4.3) \\
my_2'' + c_d y_2' + ky_2 + \gamma_d y_1 + \beta_3d(x_2^2 + y_2^2)y_2 &= 0, \quad (4.4)
\end{align*}
\]
where $\omega$, $c_d$, $k$, $\beta_{3d}$ and $\gamma_d$ are the angular velocity of the shaft, the viscous damping coefficient, the linear spring constant of the elastic shaft, the cubic nonlinear spring constant and the linear spring constant of the coupling, respectively.

### 4.1.2 Dimensionless equations of motion

The length and time are normalized using the length of a span $l$ and the inverse of linear natural frequency of a span $1/\sqrt{k/m}$. We denote the dimensionless quantities of $t$, $x_1$, $y_1$, $x_2$ and $y_2$ with $t^*$, $x_1^*$, $y_1^*$, $x_2^*$ and $y_2^*$, respectively. Hence, we obtain the dimensionless equations as follows:

\[ \ddot{x}_1 + c\dot{x}_1 + x_1 + \gamma x_2 + \beta_3(x_1^2 + y_1^2)x_1 = e\nu^2 \cos \nu t \]  
\[ \ddot{y}_1 + c\dot{y}_1 + y_1 + \gamma y_2 + \beta_3(x_1^2 + y_1^2)y_1 = e\nu^2 \sin \nu t \]  
\[ \ddot{x}_2 + c\dot{x}_2 + x_2 + \gamma x_1 + \beta_3(x_2^2 + y_2^2)x_2 = 0 \]  
\[ \ddot{y}_2 + c\dot{y}_2 + y_2 + \gamma y_1 + \beta_3(x_2^2 + y_2^2)y_2 = 0, \]

where the dimensionless parameters, $e$, $c$, $\gamma$, $\beta_3$ and $\nu$, are expressed as follows:

\[ e = \frac{c_d}{l}, \quad c = \frac{c_d}{\sqrt{mk}}, \quad \gamma = \frac{\gamma_d}{k}, \quad \beta_3 = \frac{\beta_{3d} l^2}{k}, \quad \nu = \frac{\omega}{\sqrt{k/m}}. \]

The dot denotes the derivative with respect to dimensionless time $t^*$. In Eqs. (4.5) - (4.8) and hereafter, the asterisk is omitted.
4.1.3 Transformation into complex form

Next, we rewrite the motion of the system by using complex vector representation as follows [9]:

\[ z_1 = x_1 + iy_1 \]  \hspace{1cm} (4.9)
\[ z_2 = x_2 + iy_2. \]  \hspace{1cm} (4.10)

Then, the dimensionless equations of motion are transformed as follows:

\[ \ddot{z}_1 + c\dot{z}_1 + z_1 + \gamma z_2 + \beta_3 |z_1|^2 z_1 = e\nu^2 e^{i\nu t} \]  \hspace{1cm} (4.11)
\[ \ddot{z}_2 + c\dot{z}_2 + z_2 + \gamma z_1 + \beta_3 |z_2|^2 z_2 = 0. \]  \hspace{1cm} (4.12)

The transformation is an essential process to separately obtain the averaged equations of the forward and backward whirling modes.

4.2 Theoretical analysis

4.2.1 Averaged equation

In this section, we derive averaged equations from the dimensionless equations of motion, Eqs. (4.11) and (4.12), by using the method of multiple scales [10]. First, we perform the scaling of some parameters as

\[ c = \epsilon^2 \hat{c}, \ \gamma = \epsilon^2 \hat{\gamma}, \ e = \epsilon^3 \hat{e}, \]

where
where (·) denotes “of order O(1)” and $\epsilon(\epsilon \ll 1)$ is a bookkeeping device. Then, the dimensionless equations of motion are

\[
\ddot{z}_1 + \epsilon^2 \dot{c} \dot{z}_1 + z_1 + \epsilon^2 \gamma z_2 + \beta_3 |z_1|^2 z_1 = \epsilon^3 c \nu^2 e^{i \nu t} \tag{4.13}
\]

\[
\ddot{z}_2 + \epsilon^2 \dot{c} \dot{z}_2 + z_2 + \epsilon^2 \gamma z_1 + \beta_3 |z_2|^2 z_2 = 0. \tag{4.14}
\]

We seek the approximate solutions of Eqs. (4.13) and (4.14) in the form

\[
z_1 = \epsilon z_{11} + \epsilon^3 z_{13} + \cdots \tag{4.15}
\]

\[
z_2 = \epsilon z_{21} + \epsilon^3 z_{23} + \cdots. \tag{4.16}
\]

We introduce the multiple time scales as follows:

\[
t_0 = t, \ t_2 = \epsilon^2 t,
\]

where $t_0$ is fast time scale, and $t_1$ is slow time scale.

Also, to express quantitatively the nearness of the rotational speed $\nu$ to the natural frequency of the rotor, we introduce a detuning parameter $\sigma$ defined by

\[
\nu = 1 + \sigma = 1 + \epsilon^2 \tilde{\sigma}.
\]

Substituting Eqs. (4.15) and (4.16) into Eqs. (4.13) and (4.14) and equating coefficients of like powers of $\epsilon$ yields
\[ O(\epsilon) \]

\[ D_0^2 z_{11} + z_{11} = 0 \]  
\[ D_0^2 z_{21} + z_{21} = 0 \]  
\[ (4.17) \]

\[ O(\epsilon^3) \]

\[ D_0^2 z_{13} + z_{13} = -2D_0D_2 z_{11} - \hat{c}D_0 z_{11} - \hat{\gamma}z_{21} - \beta_3 |z_{11}|z_{11} + \hat{c}e^{i\nu t_0} \]  
\[ D_0^2 z_{23} + z_{23} = -2D_0D_2 z_{21} - \hat{c}D_0 z_{21} - \hat{\gamma}z_{11} - \beta_3 |z_{21}|z_{21}, \]  
\[ (4.19) \]

where \( D_i = \partial / \partial t_i \).

The solutions of Eqs. (4.17) and (4.18) can be written as

\[ z_{11} = A_{1f} e^{i\nu t_0} + A_{1b} e^{-i\nu t_0} \]  
\[ (4.21) \]

\[ z_{21} = A_{2f} e^{i\nu t_0} + A_{2b} e^{-i\nu t_0}, \]  
\[ (4.22) \]

where \( A_{1f} \) and \( A_{1b} \) are complex amplitudes of the forward and backward whirls of the rotor 1, respectively. Similarly, \( A_{2f} \) and \( A_{2b} \) are complex amplitudes of the forward and backward whirls of the rotor 2, respectively. These complex amplitudes are varied with slow time scale \( t_2 \).
Substituting Eqs. (4.21) and (4.22) into Eqs. (4.19) and (4.20) gives

\[ D_0^2 z_{13} + z_{13} = \{ -2iD_2 A_{1f} - i\hat{c}A_{1f} - \hat{\gamma}A_{2f} - \beta_3(|A_{1f}|^2 + 2|A_{1b}|^2)A_{1f} + \hat{c}e^{i\sigma t_2} \} e^{it_0} \]

\[ + \{ 2iD_2 A_{1b} + i\hat{c}A_{1b} - \hat{\gamma}A_{2b} - \beta_3(|A_{1b}|^2 + 2|A_{1f}|^2)A_{1b} \} e^{-it_0} + \text{N.S.T.} \]

(4.23)

\[ D_0^2 z_{23} + z_{23} = \{ -2iD_2 A_{2f} - i\hat{c}A_{2f} - \hat{\gamma}A_{1f} - \beta_3(|A_{2f}|^2 + 2|A_{2b}|^2)A_{2f} \} e^{it_0} \]

\[ + \{ 2iD_2 A_{2b} + i\hat{c}A_{2b} - \hat{\gamma}A_{1b} - \beta_3(|A_{2b}|^2 + 2|A_{2f}|^2)A_{2b} \} e^{-it_0} + \text{N.S.T.} \]

(4.24)

where N.S.T. denotes terms not to proportional to \( e^{it_0} \) or \( e^{-it_0} \). The condition not to produce the secular term proportional to \( e^{it_0} \) in the solution of \( z_{13} \) is

\[ 2iD_2 A_{1f} + i\hat{c}A_{1f} + \hat{\gamma}A_{2f} + \beta_3(|A_{1f}|^2 + 2|A_{1b}|^2)A_{1f} - \hat{c}e^{i\sigma t_2} = 0. \]

(4.25)

Also, the condition not to produce the secular term proportional to \( e^{-it_0} \) is

\[ 2iD_2 A_{1b} + i\hat{c}A_{1b} - \hat{\gamma}A_{2b} - \beta_3(|A_{1b}|^2 + 2|A_{1f}|^2)A_{1b} = 0. \]

(4.26)

Generally in the case of rectilinear systems the complex conjugate of the solvability condition Eq. (4.25) is equivalent to the solvability condition Eq. (4.26). One of them is needed to obtain the averaged equation. However, contrast with the rectilinear systems, these conditions are generally independent and the first and second conditions lead to averaged equations of the forward and backward whirling modes, respectively. Similarly, the following
conditions not to produce the secular terms from $z_{23}$ are averaged equations for the forward and backward whirling modes of the rotor 2.

\begin{equation}
2iD_2A_{2f} + icA_{2f} + \gamma A_{1f} + \beta_3(|A_{2f}|^2 + 2|A_{2b}|^2)A_{2f} = 0 \tag{4.27}
\end{equation}

\begin{equation}
2iD_2A_{2b} + icA_{2b} - \gamma A_{1b} - \beta_3(|A_{2b}|^2 + 2|A_{2f}|^2)A_{2b} = 0. \tag{4.28}
\end{equation}

Substituting

\begin{align*}
A_{1f} &= a_{1f}e^{i(\varphi_{1f} + \sigma t_2)} \\
A_{1b} &= a_{1b}e^{i(\varphi_{1b} - \sigma t_2)} \\
A_{2f} &= a_{2f}e^{i(\varphi_{2f} + \sigma t_2)} \\
A_{2b} &= a_{2b}e^{i(\varphi_{2b} - \sigma t_2)}
\end{align*}

into Eqs. (4.25)-(4.28), and separating real and imaginary parts, we obtain the following averaged equations expressing the slow time scale modulations of the amplitudes and phases of forward and backward whirls in each rotor:

\begin{align*}
\frac{da_{1f}}{dt} &= -\frac{1}{2}ca_{1f} + \frac{1}{2}\gamma a_{2f} \sin(\varphi_{1f} - \varphi_{2f}) - \frac{1}{2} \epsilon \sin \varphi_{1f} \tag{4.29} \\
\frac{d\varphi_{1f}}{dt} &= -\sigma a_{1f} + \frac{1}{2}\gamma a_{2f} \cos(\varphi_{1f} - \varphi_{2f}) + \frac{1}{2} \beta_3 a_{1f}^2 + \beta_3 a_{1f}a_{1b}^2 - \frac{1}{2} \epsilon \cos \varphi_{1f} \tag{4.30} \\
\frac{da_{1b}}{dt} &= -\frac{1}{2}ca_{1b} - \frac{1}{2}\gamma a_{2b} \sin(\varphi_{1b} - \varphi_{2b}) \tag{4.31} \\
\frac{d\varphi_{1b}}{dt} &= \sigma a_{1b} - \frac{1}{2}\gamma a_{2b} \cos(\varphi_{1b} - \varphi_{2b}) - \frac{1}{2} \beta_3 a_{1b}^2 - \beta_3 a_{1f}^2 a_{1b} \tag{4.32} \\
\frac{da_{2f}}{dt} &= -\frac{1}{2}ca_{2f} - \frac{1}{2}\gamma a_{1f} \sin(\varphi_{1f} - \varphi_{2f}) \tag{4.33} \\
\frac{da_{2b}}{dt} &= -\frac{1}{2}ca_{2b} + \frac{1}{2}\gamma a_{1b} \sin(\varphi_{1b} - \varphi_{2b}) \tag{4.34}
\end{align*}
\[
a_{2f} \frac{d\varphi_{2f}}{dt} = -\sigma a_{2f} + \frac{1}{2} \gamma a_{1f} \cos(\varphi_{1f} - \varphi_{2f}) + \frac{1}{2} \beta_3 a_{2f}^3 + \beta_3 a_{2f} a_{a_{2b}}^2 \quad (4.34)
\]
\[
a_{a_{2b}} \frac{da_{2b}}{dt} = -\frac{1}{2} \gamma a_{1b} \sin(\varphi_{1b} - \varphi_{2b}) \quad (4.35)
\]
\[
a_{2b} \frac{d\varphi_{2b}}{dt} = \sigma a_{2b} - \frac{1}{2} \gamma a_{1b} \cos(\varphi_{1b} - \varphi_{2b}) - \frac{1}{2} \beta_3 a_{2b}^3 - \beta_3 a_{2f} a_{a_{2b}} \quad (4.36)
\]

where \( a_{1f} = \epsilon \hat{a}_{1f}, \ a_{1b} = \epsilon \hat{a}_{1b}, \ a_{2f} = \epsilon \hat{a}_{2f} \) and \( a_{2b} = \epsilon \hat{a}_{2b} \). Therefore, we obtain the approximate solutions of Eqs. (4.11) and (4.12) as follows:

\[
z_1 = a_{1f} e^{i(\nu t + \varphi_{1f})} + a_{1b} e^{i(-\nu t + \varphi_{1b})} + O(\epsilon^3) \quad (4.37)
\]
\[
z_2 = a_{2f} e^{i(\nu t + \varphi_{2f})} + a_{2b} e^{i(-\nu t + \varphi_{2b})} + O(\epsilon^3). \quad (4.38)
\]

Next, in order to examine the stabilities of trivial fixed points \((a_b = 0)\), we express the complex amplitudes of forward and backward whirls in each rotor by using the cartesian forms as follows [12]:

\[
A_{1f} = x_{1f} + iy_{1f}, \ A_{1b} = x_{1b} + iy_{1b}
\]
\[
A_{2f} = x_{2f} + iy_{2f}, \ A_{2b} = x_{2b} + iy_{2b}.
\]

Then, the averaged equations expressing the modulations of the amplitudes in the \( x \) and \( y \) directions of forward and backward whirls in each rotor:

\[
\frac{dx_{1f}}{dt} = \sigma y_{1f} - \frac{1}{2} \gamma x_{1f} - \frac{1}{2} \beta_3 (x_{1f}^2 + y_{1f}^2) y_{1f} - \beta_3 (x_{1b}^2 + y_{1b}^2) y_{1f} \quad (4.39)
\]
\[
\frac{dy_{1f}}{dt} = -\sigma x_{1f} - \frac{1}{2} \gamma y_{1f} + \frac{1}{2} \beta_3 (x_{1f}^2 + y_{1f}^2) x_{1f} + \beta_3 (x_{1b}^2 + y_{1b}^2) x_{1f} - \frac{1}{2} \epsilon \quad (4.40)
\]
The dynamics of the system is described by only radii of rotors 1 and 2 because the forward whirling motion is only produced and the trajectories of the rotor equal to zero in the steady state. Then, the equivalent degree of freedom can be regarded 2 because the forward whirling motion is only produced and the trajectories of the rotor 1 and rotor 2 are circular. The dynamics of the system is described by only radii of the rotors 1 and 2.

Therefore, we obtain the approximate solutions of Eqs. (4.11) and (4.12) as follows:

\[
\begin{align*}
  z_1 &= x_{1f} + x_{1b} + i(y_{1f} + y_{1b}) + O(\epsilon^3) \\
  z_2 &= x_{2f} + x_{2b} + i(y_{2f} + y_{2b}) + O(\epsilon^3).
\end{align*}
\]  

### 4.2.2 Nonlinear normal modes

The steady state motion occurs when \( da_{1f}/dt = da_{1b}/dt = da_{2f}/dt = da_{2b}/dt = d\varphi_{1f}/dt = d\varphi_{1b}/dt = d\varphi_{2f}/dt = d\varphi_{2b}/dt = 0 \) and \( dx_{1f}/dt = dy_{1f}/dt = dx_{1b}/dt = dy_{1b}/dt = dx_{2f}/dt = dy_{2f}/dt = dx_{2b}/dt = dy_{2b}/dt = 0 \). Substituting these conditions into Eqs. (4.39)-(4.46) yields that the amplitudes of the backward whirling motions of the rotor 1 and rotor 2 are equal to zero in the steady state. Then, the equivalent degree of freedom can be regarded as 2 because the forward whirling motion is only produced and the trajectories of the rotor 1 and rotor 2 are circular. The dynamics of the system is described by only radii of the rotors 1 and 2.
First, to seek the nonlinear normal modes, we set $e = c = \sigma = 0$, and $d/dt = 0$. The relationship between the radii of the forward whirling motions of the rotor 1 and rotor 2 is retained by the following equation:

$$
(a_1^2 f - a_2^2 f) \left( \beta_3 - \frac{\gamma}{a_1 f a_2 f} \right) = 0.
$$

The solutions, $a_1 f = a_2 f$ and $a_1 f = -a_2 f$, correspond to in-phase and antiphase, respectively. Furthermore, this system possesses additional modes and it results that the number of modes can exceed the equivalent degree of freedom of the two-span rotor system. The additional nonlinear normal modes satisfying the equation of $\beta_3 - \gamma/a_1 f a_2 f = 0$ bifurcate from the antiphase mode as shown in Figs. 4.3 and 4.4. One of the bifurcated modes has very small ratio of $a_2 f/a_1 f$; hereafter we call this mode “anti-1” mode, and the other one has very large ratio of $a_2 f/a_1 f$; hereafter we call this mode “anti-2” mode. The anti-1 mode, i.e., the combination of (anti-1) in Figs. 4.3 and 4.4 expresses the mode localization to the rotor 1, and the anti-2 mode, i.e., the combination of (anti-2) in Figs. 4.3 and 4.4 expresses the mode localization to the rotor 2. Because the bifurcation point is $a_1 f = -a_2 f = \sqrt{-\gamma/\beta_3}$, the bifurcation occurs at higher amplitude with larger coupling stiffness $\gamma$ or smaller cubic nonlinear coefficient $\beta_3$. This fact is also found from the comparison between Figs. 4.3 and 4.4, and 4.5 and 4.6. It is well known that the frequency response curves exist in the neighborhood of backbone curves based on the nonlinear normal modes. In the next section, comparing with the above obtained backbone curves, we characterize frequency response curves of the two-span rotor system with weak coupling and nonlinearity.
Figure 4.3: Nonlinear normal modes bifurcated from the antiphase mode \( (\beta_3 = 2.52 \times 10^3, \gamma = -1.09 \times 10^{-2}) \)

Figure 4.4: Nonlinear normal modes bifurcated from the antiphase mode \( (\beta_3 = 2.52 \times 10^3, \gamma = -1.09 \times 10^{-2}) \)
Figure 4.5: Nonlinear normal modes bifurcated from the antiphase mode ($\beta_3 = 2.52 \times 10^3$, $\gamma = -8.72 \times 10^{-2}$)

Figure 4.6: Nonlinear normal modes bifurcated from the antiphase mode ($\beta_3 = 2.52 \times 10^3$, $\gamma = -8.72 \times 10^{-2}$)
4.2.3 Frequency response curves and mode localizations

We reconsider the conditions in Eqs. (4.29)-(4.36) in the steady states. Furthermore, examining their stabilities by these Equations [11], we obtain frequency response curves as Figs. 4.7 and 4.8, where $c = 5.35 \times 10^{-4}$, $\gamma = -8.72 \times 10^{-2}$, $\beta_3 = 2.52 \times 10^3$ and $e = 3.85 \times 10^{-5}$. The solid and dashed lines denote stable and unstable steady state amplitude, respectively. The frequency response curves exist along the backbone curves. In particular, in the neighborhood of the bifurcation point on the backbone curve, the frequency response curves become complex as shown in Figs. 4.9 and 4.10.

Figures 4.11 and 4.12 show the frequency response curves in the case of that the damping coefficient is increased to $c = 2.14 \times 10^{-3}$ and the coupling effect is decreased to $\gamma = -1.09 \times 10^{-2}$ from those in the case of Figs. 4.7 and 4.8, respectively. The solid and dashed lines denote stable and unstable steady state amplitude, respectively. As shown in the previous section, the decrease of the coupling effect makes the amplitude at the bifurcation point of the nonlinear normal modes smaller. Therefore, the frequency response curve has complex shape in the region of small amplitude compared with that in Figs. 4.7 and 4.8. On the other hand, with increasing the damping, the branch of the frequency response curve ($\ast 2$) along the additional antiphase mode (anti-2 mode) bifurcated on the antiphase mode becomes shorter. Hence, the region of the rotational speed, such that the forced response is localized in the neighborhood of the additional antiphase mode (anti-2 mode), becomes narrow as the effect of damping increases; it is harder that the forced oscillation is localized to the rotor 2 without unbalance. On the other hand, the branch of the frequency response curve ($\ast 1$) along the additional antiphase mode (anti-1 mode) bifurcated on the antiphase
mode is not affected by the increase of damping and does not becomes much shorter; it is
easy that the forced oscillation is localized to the rotor 1 with unbalance.

Figure 4.7: Frequency response curve ($c = 5.35 \times 10^{-4}, \gamma = -8.72 \times 10^{-2}, \beta_3 = 2.52 \times 10^3,$
$e = 3.85 \times 10^{-5},$ —— : stable, - - - - : unstable)

Figure 4.8: Frequency response curve ($c = 5.35 \times 10^{-4}, \gamma = -8.72 \times 10^{-2}, \beta_3 = 2.52 \times 10^3,$
$e = 3.85 \times 10^{-5},$ —— : stable, - - - - : unstable)
Figure 4.9: Frequency response curve (expansion of Fig. 4.7, $0.05 \leq \sigma \leq 0.15$, $4 \times 10^{-3} \leq (a_{1f}, a_{2f}) \leq 8 \times 10^{-3}$, —— : stable, - - - - : unstable)

Figure 4.10: Frequency response curve (expansion of Fig. 4.8, $0.05 \leq \sigma \leq 0.15$, $4 \times 10^{-3} \leq (a_{1f}, a_{2f}) \leq 8 \times 10^{-3}$, —— : stable, - - - - : unstable)
Figure 4.11: Frequency response curve \((c = 2.14 \times 10^{-3}, \gamma = -1.09 \times 10^{-2}, \beta_3 = 2.52 \times 10^3, \epsilon = 3.85 \times 10^{-5}, \quad ---: \text{stable}, \quad - - - -: \text{unstable})\)

Figure 4.12: Frequency response curve \((c = 2.14 \times 10^{-3}, \gamma = -1.09 \times 10^{-2}, \beta_3 = 2.52 \times 10^3, \epsilon = 3.85 \times 10^{-5}, \quad ---: \text{stable}, \quad - - - -: \text{unstable})\)
Figures 4.13-4.15 express the trajectories of rotors at the rotor speeds shown with the symbols □, △ and ○ in Figs. 4.11 and 4.12. At the condition of the symbol □, where the nonlinear normal modes is not bifurcated, the amplitude of rotor 1 is almost equal to that of rotor 2 as shown in Fig. 4.13. At the symbol △, the amplitude of rotor 1 is much larger than that of rotor 2 as shown in Fig. 4.14. The usage of this phenomenon prevents the influence of the oscillation caused by the unbalance of rotor 1 on the rotor 2. At the symbol ○, even though the rotor 1 has unbalance, the amplitude of rotor 2 is much larger than that of rotor 1 as shown in Fig. 4.15. Due to the occurrence of such a response, there is a possibility of wrong diagnosis that the rotor 2 has unbalance. On the other hand, this phenomenon indicates that the rotor 2 can be utilized as a dynamic vibration absorber for the rotor 1.
Figure 4.13: Theoretical orbit without localization ($\sigma = 0.0015$, at the symbol □ in Figs. 4.11 and 4.12)

Figure 4.14: Localization of the rotor 1 with unbalance ($\sigma = 0.0597$, at the symbol □ in Figures 4.11 and 4.12)

Figure 4.15: Localization of the rotor 2 without unbalance ($\sigma = 0.0207$, at the symbol ○ in Figs. 4.11 and 4.12)
Chapter 5

Experiments

5.1 Experimental setup

Figure 5.1 shows the experimental setup. Two elastic shaft with circular cross section whose length and diameter are \( l = 0.708 \text{ m} \) and \( 1.2 \times 10^{-2} \text{ m} \), respectively. They are supported at both ends by a self-aligning double-ball bearing (JIS \( \#1200 \)) and a single-row deep groove ball bearing (JIS \( \#6204 \)). A disk mounted at the center of the shaft has the diameter of 0.3 m and mass of 8.03 kg. The two rotors are coupled by spring in imitation of a flange type shaft coupling. The shaft is driven by the three-phase induction motor (Meidensha Corp., TIS85–NR) through V-belt and V-pulley. The lateral and vertical motions of disks and the angular velocities are measured by laser sensors (KEYENCE LX2-02 and LX2-V10) and the rotary encoders (Ono-Sokki, RP-432Z), respectively.
Figure 5.1: Experimental setup (two-span rotor system)
\[
\begin{align*}
e &= \frac{e_d}{l} = \frac{3.9 \times 10^{-6}}{0.708} = 5.51 \times 10^{-6} \\
c_x &= \frac{c_{xd}}{\sqrt{mk}} = \frac{3.50}{\sqrt{8.03 \times 3.50 \times 10^4}} = 6.60 \times 10^{-3} \\
c_y &= \frac{c_{yd}}{\sqrt{mk}} = \frac{1.58}{\sqrt{8.03 \times 3.50 \times 10^4}} = 2.98 \times 10^{-3} \\
\gamma &= \frac{\gamma_d}{k} = \frac{-358}{3.50 \times 10^4} = -1.02 \times 10^{-2} \\
\beta_3 &= \frac{\beta_{3d} l^2}{k} = \frac{6.99 \times 10^8 \times (0.708)^2}{3.50 \times 10^4} = 1.00 \times 10^4 \\
\omega &= \frac{\sqrt{k/m}}{\sqrt[5]{3.50 \times 10^4 / 8.03}} \\
\nu &= \frac{\omega}{\sqrt{k/m}} = \frac{\omega}{\sqrt[5]{3.50 \times 10^4 / 8.03}} 
\end{align*}
\]

5.2 Identification of spring constants of rotor

Prior to the experiments, we experimentally obtain the linear spring constant of the rotor \(k\), the cubic nonlinear spring constant \(\beta_{3d}\) and the quintic nonlinear spring constant \(\beta_{5d}\).

Figure 5.2 shows the load-deflection curves in the \(y\) direction. The circles in this figure denote the experimental one. It is easy to see that this rotor has nonlinear stiffness.

Next, using these experimental data, we find a least-squares fit to this curve

\[
W = ky + \beta_{3d} y^3 + \beta_{5d} y^5. \tag{5.1}
\]

Then, the parameter’s values are

\[
k = 3.09 \times 10^4 \text{ N/m, } \beta_{3d} = 1.58 \times 10^9 \text{ N/m}^3, \beta_{5d} = -3.86 \times 10^{13} \text{ N/m}^5, \]

\[
y_{st} = -2.11 \times 10^{-3} \text{ m.}
\]
Similarly, we find a least-squares fit to this curve from the experimental data

\[ W = ky + \beta_3dy^3. \]  \hspace{1cm} (5.2)

Then, we obtain the parameter’s values as follows:

\[ k = 3.50 \times 10^4 \text{ N/m}, \quad \beta_3 = 6.99 \times 10^8 \text{ N/m}^3, \quad y_{st} = -2.07 \times 10^{-3} \text{ m}. \]

The solid line in Fig. 5.2 is the load-deflection curve obtained from Eq. (5.1). The result shows that the theoretical load-deflection curve quantitatively in good agreement with the experimental one. The solid line in Fig. 5.3 is the load-deflection curve obtained from Eq. (5.2).
5.3 Experimental results

5.3.1 Single-span rotor system

In the experiment, we measure deflections in the $x$ and $y$ directions from the initial equilibrium position quasistatic sweep passaging through the major critical speed. In Fig. 5.4, we show the experimental and theoretical frequency response curves of the single-span rotor.
Figure 5.4: Experimental and theoretical frequency response curves

in the case of $c_d = 6.59 \times 10^{-3}$ mm, $c_{xd} = 3.50$ N · s/m, $c_{yd} = 1.58$ N · s/m. As shown in Fig. 5.4(a), the response in the $x$ direction is bend to the right and exhibits the nonlinear feature of hardening spring. On the other hand, the response in the $y$ direction is bend to the left and exhibits the nonlinear feature of softening spring due to the effect of gravity, as shown in Fig. 5.4(b). Also, it is shown that the peak in the $x$ direction is lower than one in the $y$ direction by the difference of the damping in the $x$ and $y$ directions. These experimental results are qualitatively agreement with the theoretical ones, as shown in Fig. 5.4. It is theoretically and experimentally concluded that it is necessary to consider up to quintic nonlinearity in this system.

3.2 Two-span rotor system

Figures 5.5 and 5.6 show the frequency response curves of the two-span rotor system in the case of $e_d = 3.9 \times 10^{-3}$ mm. When the angular velocity of the shaft $\omega$ is near the natural frequency in the $x$ direction $\omega_{xd}$, the amplitudes in the $x$ direction tend to increase as shown in Fig. 5.5. On the other hand, when $\omega$ is near the natural frequency in the $y$ direction $\omega_{yd}$,
the amplitudes in the $y$ direction tend to increase as shown in Fig. 5.6 and these responses are softening-type ones. We can not confirm the occurrences of theoretically discussed mode localizations. This indicates that it may be necessary to reconsider the coupling structure between each span.

Figure 5.5: Experimental frequency response curve

Figure 5.6: Experimental frequency response curve
Chapter 6

Conclusions

In this research, we propose a general method to theoretically analyze nonlinear in the rotor system. Also, it is theoretically and experimentally clarified that a horizontally supported single-span rotor with cubic and quintic nonlinearities occurs hardening and softening types responses depending on the rotational speed, due to the effects of gravity and nonlinear characteristics of the rotor.

Furthermore, we investigate the nonlinear normal modes in a two-span rotor system in which a balanced rotor and an unbalanced rotor are weakly coupled. First, the averaged equations are derived by the method of multiple scales. The bifurcation analysis of the nonlinear normal mode is performed and the backbone curve is obtained. Also, frequency response curve with respect to the rotational speed has complex shape in the neighborhood of the bifurcation points of the backbone curve. As a result, two different kinds of mode localizations are analytically predicted; the whirling motion caused by the unbalance rotor is localized to the same rotor with unbalance and in the other case, the whirling motion is
localized to the other rotor without unbalance.
Bibliography


Related Presentation


(Presentation at the 11th Kanto Branch Conference of the Japan Society of Mechanical Engineers, March 18-19, 2005, Tokyo Metropolitan University, Tokyo)

• Kunitoh, Y., Yabuno, H., Inoue, T., and Ishida, Y., Nonlinear Normal Modes in Flexibly Coupled Two Rotors (Symmetry Breaking due to the Gravity), Proc. of D&D 2005, No. 132.  
(Presentation at Dynamics and Design Conference 2005, August 27-30, 2005, TOKI MESSE Niigata Convention Center, Niigata)

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